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# Lorentz and Polarization Correction for the Buerger Precession Method

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The reciprocal of the product of the Lorentz factor and the polarization factor is calculated for the zero level and $\bar{\mu}=30°$.

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IN a previous publication[1] it was shown that in the precession method[2] the angular velocity with which the reciprocal lattice moves through the sphere of reflection is not constant and equal to the angular velocity of precession, but rather varies with the position of the precession axis. It attains its maximum or minimum value whenever this axis passes, respectively, through a vertical or a horizontal plane. The Lorentz factor was derived to be of the form:

$$L(\xi,\zeta,\tau)\sim\frac{1}{\Omega\sin\Upsilon\sin\bar{\nu}\sin\bar{\mu}\cos\bar{\mu}}\times\left[\frac{1}{1+\tan^2\bar{\mu}\sin^2(\tau+\eta)}+\frac{1}{1+\tan^2\bar{\mu}\sin^2(\tau-\eta)}\right],\quad(1)$$

where $\xi$ is the radial, $\zeta$ the axial, and $\tau$ the angular cylindrical coordinate of the reciprocal lattice point $P$ considered, and where the crystal precesses with an angular velocity $\Omega$ at a precession angle $\bar{\mu}$. The angle $\bar{\nu}$ is half the opening angle of the diffraction cone, while $\Upsilon$ is the projection onto the 0-level of the reciprocal lattice of the angle enclosed by incident and reflected beam. The angle $\eta$ finally describes the passage of the point $P(\xi,\zeta,\tau)$ through the surface of the sphere of reflection. The angles $\Upsilon$ and $\eta$ are related to $\xi$, $\bar{\mu}$, and $\bar{\nu}$ by the expressions,

$$\cos\Upsilon=\frac{\sin^2\bar{\mu}+\sin^2\bar{\nu}-\xi^2}{2\sin-\sin\bar{\nu}},\quad(2)$$

$$\cos\eta=\frac{\xi^2+\sin^2\bar{\mu}-\sin^2\bar{\nu}}{2\xi\sin\bar{\mu}}.\quad(3)$$

It is convenient to combine the Lorentz factor with the polarization factor,

$$p=(1+\cos^2 2\vartheta)/2.\quad(4)$$

The chart of Fig. 1 represents the reciprocal $(L.p)^{-1}$ of the product of these two factors for the case $\bar{\mu}=\bar{\nu}=30°$ (zero-level), which is probably the most useful setting for the precession instrument.

The chart was constructed in the following way. For the zero-level $\sin\vartheta=\xi/2$, so that the polarization factor may be given the form,

$$p=(8-4\xi^2+\xi^4)/8.\quad(4')$$

Furthermore, (2) reduces to

$$\cos\Upsilon=1-\tfrac{1}{2}(\xi/\sin\bar{\mu})^2,\quad(2')$$

so that in (1)

$$\sin\Upsilon\sin^2\bar{\mu}=\xi[\sin^2\bar{\mu}-(\xi/2)^2]^{\frac{1}{2}}.\quad(5)$$

For the zero level the reciprocal of the Lorentz polarization factor is therefore proportional to

$$(1/L.p)\sim A(\xi,\bar{\mu})B(\xi,\tau,\bar{\mu}),\quad(6)$$

where

$$A(\xi,\bar{\mu})=\frac{\xi[\sin^2\bar{\mu}-(\xi/2)^2]^{\frac{1}{2}}\cos\bar{\mu}}{8-4\xi^2+\xi^4},\quad(7)$$

$$B(\xi,\tau,\bar{\mu})=\left[\frac{1}{1+\tan^2\bar{\mu}\sin^2(\tau+\eta)}+\frac{1}{1+\tan^2\bar{\mu}\sin^2(\tau-\eta)}\right]^{-1},\quad(8)$$

and where $\eta$ is related to $\xi$ and $\bar{\mu}$ by the equation,

$$\cos\eta=\tfrac{1}{2}\xi\sin\bar{\mu}.\quad(3')$$

The following expansion, valid for $\bar{\mu}=30°$ ($\tan\bar{\mu}=\frac{1}{3}$) proved convenient for the evaluation of $B(\xi,\tau,30°)$:

$$\left[\frac{1}{1+\frac{1}{3}\sin^2(\tau+\eta)}+\frac{1}{1+\frac{1}{3}\sin^2(\tau-\eta)}\right]^{-1}=\frac{1}{2}\left[\frac{7}{6}-\frac{1}{6}\cos2\tau\cos2\eta-\frac{1}{6\cdot7}\sin^2 2\tau\sin^2 2\eta-\frac{1}{6\cdot7\cdot7}\cos2\tau\sin^2 2\tau\cos2\eta\sin^2 2\eta-\frac{1}{6\cdot7\cdot7\cdot7}\cos^2 2\tau\sin^2 2\tau\cos^2 2\eta\sin^2 2\eta-\cdots\right].\quad(9)$$

For $\tau=0°$ and $\tau=90°$ this expansion breaks off after the second term, for $\tau=45°$ after the third term, while the second term vanishes; and, in general, the first three terms represent $B(\xi,\tau,30°)$ to better than 0.35 percent, which was deemed sufficiently accurate for the

[1] J. Waser, Rev. Sci. Instr. **22**, 563 (1951).
[2] M. J. Buerger, *The Photography of the Reciprocal Lattice* (Am. Soc. for X-ray and Electron Diffraction, 1944).

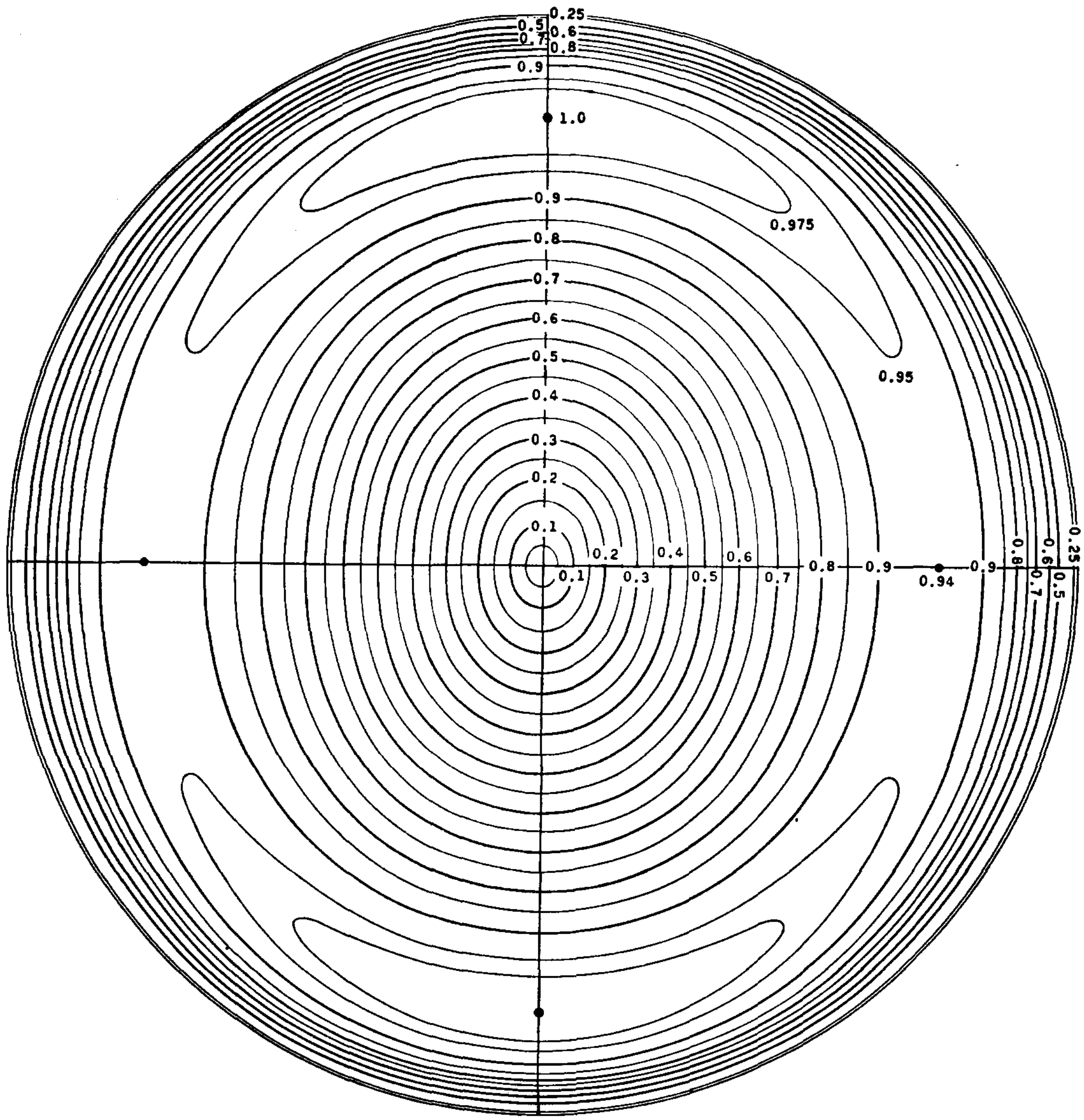


FIG. 1. Lorentz polarization correction for zero-level and $\bar{\mu}=0°$.

present calculations. The values of $A(\xi, 30°)$ have been tabulated by Evans, Tilden, and Adams.[3] The first 3 terms of (9) were accordingly evaluated at constant $\tau$ for various values of $\xi$, using relations (3′) to find $\eta$ for a given $\xi$. The results were multiplied with the corresponding values of $A(\xi, 30°)$ and the products plotted in dependence of $\xi$ after suitable normalization so as to make the maximum value of $(\mathrm{L.p.})^{-1}$ equal to 1.00. From such graphs constructed for a selected group of $\tau$-values the $\xi$-values were found at which the normalized $(\mathrm{L.p.})^{-1}$ was equal to integral multiples of 0.05. The results of this procedure were used to obtain the chart shown in Fig. 1, which contains the Lorentz polarization correction in dependence of the reciprocal lattice coordinates of the zero level. After suitable scale transformation which may be done photographically the chart can be used directly with a zero-level precession photograph. The outermost circle of the chart corresponds to $\xi=1.00$, and its orientation is the same as that of the film in the cassette of the precession instrument.

[3] Evans, Tilden, and Adams, Rev. Sci. Instr. **20**, 155 (1949).